Q1a

a)

$(2-x)(3x+1)$

$(6x+2-3x^2-x)(x+1)$

$(5x+2-3x^2)(x+1)$

$5x^2+2x-3x^3$

$-3x^2+5x\quad+2$

$$\boxed{2x^2+7x-3x^3+2}$$

Q1b

ALTERNATE SOLUTION

$c^2=2(5x-2y+3)^2$

$\sqrt{\phantom{x}}\qquad\sqrt{\phantom{x}}$

$c=\sqrt{2(5x-2y+3)^2}$

$c=\pm(5x-2y+3)\sqrt{2}$

LENGTH CAN ONLY BE POSITIVE

$$\boxed{c=(5x-2y+3)\sqrt{2}\text{ UNITS}}$$

EITHER SOLUTION IS A VALID ANSWER

b)

PYTHAGORAS $a^2+b^2=c^2$

$$a^2+a^2=c^2$$

$$2a^2=c^2$$

$$c^2=2(5x-2y+3)^2$$

EXPANDING BRACKETS

$$2(5x-2y+3)(5x-2y+3)$$

$$25x^2-10xy+15x$$
$$-10xy+4y^2-6y$$
$$+15x-6y+9$$

$$c^2=2(25x^2-20xy+30x+4y^2-12y+9)$$

$$c^2=50x^2-40xy+60x+8y^2-24y+18$$

$$\boxed{c=\sqrt{50x^2-40xy+60x+8y^2-24y+18}\ \text{UNITS}}$$

Q2

$(2x-3y)(2x-3y)(y-2x)$

$(4x^2-6xy-6xy+9y^2)(y-2x)$

$(4x^2-12xy+9y^2)(y-2x)$

$y(4x^2-12xy+9y^2)$

$-2x(4x^2-12xy+9y^2)$

$4x^2y-12xy^2+9y^3$

$-8x^3+24x^2y-18xy^2$

$-8x^3+28x^2y-30xy^2+9y^3$

$\boxed{a=-8 \quad b=28 \quad c=-30 \quad d=9}$

Q3

$$x(\overbrace{15x^2 + 19x - 10}^{-150})$$

$$(15x - 6)(15x + 25)$$

$$\div 3 \qquad \div 5$$

$$x(5x - 2)(3x + 5)$$

$-150$

$10 \quad 15$

$3 \quad 50$

$5 \quad 30$

$\boxed{-6 + 25 = 19}$

<u>OR</u>

USING CALCULATOR SOLUTIONS

$x = -\frac{5}{3} \qquad x = \frac{2}{5}$

$3x = -5 \qquad 5x = 2$

$3x + 5 = 0 \qquad 5x - 2 = 0$

$(3x + 5) \qquad (5x - 2)$

$$\boxed{x(5x - 2)(3x + 5)}$$

Q4

INSPECTION $x^3 - 19x - 30$

$-5x^2$ $-25x$

$$(x-5)(x^2+5x+6)$$

$+5x^2$ $+6x$

OR

LONG DIVISION

$$
\begin{array}{r|l}
 & x^2 + 5x + 6 \\
\hline
x-5 & \cancel{x^3} - 19x - 30 \\
 & -(\cancel{x^3} - 5x^2) \\
\hline
 & +\cancel{5x^2} - 19x - 30 \\
 & -(\cancel{5x^2} - 25x) \\
\hline
 & \cancel{6x - 30} \\
 & -(\cancel{6x - 30}) \\
\hline
 & 0
\end{array}
$$

$$\boxed{x^2+5x+6}$$

Q5a

a) $x^3 - 28x + 48$

INSPECTION

$$(x-3)(x^2+3x-19)-9$$

$-9x$ $\quad -3x^2 \quad +3x^2 \quad -19x$

$48 -$
$-3\times-19=57$
$-9$

$\boxed{\text{REMAINDER } -9}$

OR

LONG DIVISION

$$
\begin{array}{r}
x^2+3x-19 \\
x-3\ \overline{)\ \cancel{x^3}-28x+48} \\
-(\cancel{x^3}-3x^2) \\
\hline
+\cancel{3x^2}-28x+48 \\
-(\cancel{3x^2}-9x) \\
\hline
-\cancel{19x}+48 \\
-(-\cancel{19x}+57) \\
\hline
-9
\end{array}
$$

$\boxed{\text{REMAINDER } -9}$

Q5b

b) $x^3 - 28x + 48$

BY INSPECTION

$$(x+6)(x^2-6x+8)$$

$6x^2 \quad -6x^2 \quad -36x \quad +8x$

FACTORISE QUADRATIC

$$\boxed{(x+6)(x-4)(x-2)}$$

Q6a

a)

$$6x^3-19x^2+11x+6$$

$-10x^2$

$-9x^2$

$$(2x-3)(3x^2-5x-2)$$

$15x$

$-4x$

$$\boxed{(2x-3)(3x^2-5x-2)}$$

Q6b

b)

$-6$

$$(2x-3)(3x^2-5x-2)$$

FACTORISE QUADRATIC

$-6$

$1 \quad -6$

$$(3x+1)(3x-6)$$

$\div 3$

$$\boxed{(2x-3)(3x+1)(x-2)}$$

Q6c

c)

SOLVE FOR $f(x)=0$

$$(2x-3)(3x+1)(x-2)=0$$

$2x-3=0 \quad 3x+1=0 \quad x-2=0$

$$\boxed{x=\frac{3}{2} \quad x=\frac{-1}{3} \quad x=2}$$

a

a)

$(x-p)$ IS A FACTOR IF $f(p)=0$

$(2x+1)=0 \quad x=-\frac{1}{2}$

CHECK $f\left(-\frac{1}{2}\right)=0$

$$4\left(-\frac{1}{2}\right)^3-7\left(-\frac{1}{2}\right)-3=0$$

$f\left(-\frac{1}{2}\right)=0$ SO $(2x+1)$ IS A FACTOR OF $f(x)$

Q7b

b) $4x^3 - 7x - 3$

DIVIDE BY $(2x+1)$

$$(2x+1)(2x^2 - x - 3)$$

$2x^2$, $-6x$, $-x$, $-2x^2$

FACTORISE QUADRATIC

$-6$

$1 \quad -6$

$\boxed{2 \quad -3}$

$$(2x+2)(2x-3)$$

$\div 2$

$$(x+1)(2x-3)$$

$$\boxed{(2x+1)(x+1)(2x-3)}$$

Q7c

c) $(2x+1)(x+1)(2x-3) = 0$

$$\boxed{x = -\frac{1}{2} \quad x = -1 \quad x = \frac{3}{2}}$$

Q8a

[3]

a) FORM SIMULTANEOUS EQUATIONS

$$f(2)=(2)^3+r(2)^2+s(2)-30=0$$
$$8+4r+2s-30=0$$
$$① \quad 4r+2s=22$$

$$f(-3)=(-3)^3+r(-3)^2+s(-3)-30=-240$$
$$-27+9r-3s-30=-240$$
$$② \quad 9r-3s=-183$$

$$\begin{array}{lrl} ①\times 3 & 12r+6s & =66 \\ ②\times 2 \quad (+) & 18r-6s & =-366 \\ \hline & 30r & =-300 \\ & r & =-10 \end{array}$$

SUB INTO ①

$$4(-10)+2s=22$$
$$-40+2s=22$$
$$2s=62$$
$$s=31$$

CHECK ②

$$9(-10)-3(31)=-183$$
$$-90-93=-183 \checkmark\checkmark$$

$$\boxed{r=-10 \quad s=31}$$

Q8b

$r = -10 \qquad s = 31$

$f(x) = x^3 - 10x^2 + 31x - 30$

IF $f(2) = 0$, $(x-2)$ MUST BE A FACTOR

$-8x^2 \qquad +15x$

$(x-2)(x^2 - 8x + 15)$

$-2x^2$

$+16x$

FACTORISE QUADRATIC

$(x-3)(x-5)$

$$\boxed{(x-2)(x-3)(x-5)}$$

Q9a

"Extended Factor Theorem"

If $f\left(\frac{b}{a}\right)=0$, then $(ax-b)$ is a factor of $f(x)$

a) $f\left(\frac{3}{4}\right)=4\left(\frac{3}{4}\right)^3-7\left(\frac{3}{4}\right)^2-21\left(\frac{3}{4}\right)+18$

$=4\left(\frac{27}{64}\right)-7\left(\frac{9}{16}\right)-21\left(\frac{3}{4}\right)+18$

$=\frac{27}{16}-\frac{63}{16}-\frac{63}{4}+18$

$=\frac{27-63-252+288}{16}$

$=0$

So by the Factor Theorem,

$(4x-3)$ is a factor of $f(x)$,

Q9b

b) $4x^3-7x^2-21x+18=(4x-3)(x^2+kx-6)$

$4x^3\underline{-7x^2}-21x+18=4x^3+\underline{(4k-3)}x^2+(-3k-24)x+18$

$-7=4k-3 \implies k=-1$ } equate $x^2$ coefficients

can also do this factorisation by algebraic division, or by inspection

So

$4x^3-7x^2-21x+18=(4x-3)(x^2-x-6)$

$$\boxed{=(4x-3)(x-3)(x+2)}$$

Q9c

From part (b),

$4x^3 - 7x^2 - 21x + 18 = (4x-3)(x-3)(x+2)$

c) $\boxed{x = 3/4,\ 3,\ -2}$

Q10

"Extended Factor Theorem"

If $f(\frac{b}{a}) = 0$, then $(ax-b)$ is a factor of $f(x)$.

If $x = 2/5$,

$25(2/5)^3 + 55(2/5)^2 - 56(2/5) + 12$

$= \frac{8}{5} + \frac{44}{5} - \frac{112}{5} + 12 = \frac{8+44-112+60}{5} = 0$

So by the Factor Theorem, $(5x-2)$ is a factor.

$25x^3 + \underline{55}x^2 - 56x + 12 = (5x-2)(5x^2+kx-6)$

$= 25x^3 + (\underline{5k-10})x^2 + (-2k-30)x + 12$

$5k - 10 = 55 \Rightarrow k = 13$ } equate $x^2$ coefficients

Could also do this factorisation by algebraic division, or by inspection

$25x^3 + 55x^2 - 56x + 12 = 0$

$(5x-2)(5x^2+13x-6) = 0$

$(5x-2)(5x-2)(x+3) = 0$

The solutions are

$x = \frac{2}{5}, -3$

Q11a

"Extended Factor Theorem"

If $(ax-b)$ is a factor of $f(x)$, then $f\left(\frac{b}{a}\right)=0$

a) Because $(4x-5)$ is a factor,

$$4\left(\frac{5}{4}\right)^3-9\left(\frac{5}{4}\right)^2+a\left(\frac{5}{4}\right)+30=0$$

$$\frac{125}{16}-\frac{225}{16}+\frac{5a}{4}+30=0$$

$$\frac{125-225+20a+480}{16}=0$$

$$20a+380=0$$

$$\boxed{a=-19}$$

Q11b

From part (a), $a=-19$

b) $4x^3-9x^2-19x+30=(4x-5)(x^2+kx-6)$

$$=4x^3+(4k-5)x^2+(-5k-24)x+30$$

$4k-5=-9 \Rightarrow k=-1$ } equate $x^2$ coefficients

Could also do this factorisation by algebraic division, or by inspection

$$4x^3-9x^2-19x+30=(4x-5)(x^2-x-6)$$

$$\boxed{=(4x-5)(x+2)(x-3)}$$

Q12

(i)
$$\begin{array}{r} x^2+4 \\ x-2\,\overline{\big)\,x^3-2x^2+4x-3} \\ -(x^3-2x^2) \\ \hline 4x-3 \\ -(4x-8) \\ \hline 5 \end{array}$$

$\boxed{\text{The remainder is } 5}$

(ii) $f(2) = (2)^3 - 2(2)^2 + 4(2) - 3$
$= 8 - 8 + 8 - 3$

$\boxed{f(2) = 5}$

(iii) $\boxed{\text{They are equal}}$

This isn't just a coincidence!

From part (i),

$x^3 - 2x^2 + 4x - 3 = (x^2+4)(x-2) + 5$

When $x = 2$,

$(2)^3 - 2(2)^2 + 4(2) - 3 = \underbrace{(2^2+4)\underbrace{(2-2)}_{=0}}_{=0} + \underline{\underline{5}}$

Q13a

a) $\boxed{\text{The equation appears to be in the form } \dfrac{\text{dividend}}{\text{divisor}} = \text{quotient} + \dfrac{\text{remainder}}{\text{divisor}}}$

Q13b

b) If $g(x) = x+1$, then:

$$\frac{f(x)}{x+1} = 2x+3-\frac{4}{x+1}$$

$$f(x) = \left(2x+3-\frac{4}{x+1}\right)(x+1)$$

$$= (2x+3)(x+1) - 4$$

$$= 2x^2 + 2x + 3x + 3 - 4$$

$$\boxed{f(x) = 2x^2 + 5x - 1}$$

Q13c

c)

Multiplying $f(x)$ and $g(x)$ by the same constant will give the same result.

For example,

$$2f(x) = 2(2x^2+5x-1) = 4x^2+10x-2$$

$$2g(x) = 2(x+1) = 2x+2$$

Then

$$\frac{4x^2+10x-2}{2x+2} = \frac{\cancel{2}(2x^2+5x-1)}{\cancel{2}(x+1)}$$

$$= \frac{2x^2+5x-1}{x+1}$$

$$= 2x+3-\frac{4}{x+1}$$

Q14

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient} + \frac{\text{remainder}}{\text{divisor}}$$

$$\frac{x^3 + ax^2 + 4x - 1}{x+2} = x^2 - 4x + 12 + \frac{b}{x+2}$$

$$x^3 \underline{+a}x^2 + 4x \underline{-1} = \left(x^2 - 4x + 12 + \frac{b}{x+2}\right)(x+2)$$

$$= (x^2 - 4x + 12)(x+2) + b$$

$$= x^3 - 4x^2 + 12x + 2x^2 - 8x + 24 + b$$

$$= x^3 \underline{-2}x^2 + 4x + \underline{24+b}$$

Comparing coefficients,

$$\boxed{a = -2}$$

$$24 + b = -1$$

$$\boxed{b = -25}$$

Q15

$(x+4)$ is a factor $\Longleftrightarrow$ $f(-4)=0$ [Factor Theorem]

$$f(-4) = 16p - 22q + 14 = 0$$

$$\Longrightarrow 8p - 11q = -7 \quad ①$$

$\frac{f(x)}{(x+1)}$ has remainder $-12$ $\Longleftrightarrow$ $f(-1) = -12$ [Remainder Theorem]

$$f(-1) = 4p - 7q - 1 = -12$$

$$\Longrightarrow 8p - 14q = -22 \quad ②$$

① and ② are simultaneous equations

$$\Longrightarrow 3q = 15 \quad ① - ②$$

$$\Longrightarrow \boxed{q = 5, \quad p = 6}$$

Q16

$(3x+1)(ax^2+bx+c)+d = 3(x+\frac{1}{3})(ax^2+bx+c)+d$

By the Remainder Theorem, $f(-\frac{1}{3}) = d$

$\Rightarrow d = 3(-\frac{1}{3})^3 + 16(-\frac{1}{3})^2 - 22(-\frac{1}{3}) = -\frac{1}{9} + \frac{16}{9} + \frac{66}{9} = 9$

$(3x+1)(ax^2+bx+c)+9 =$

$3ax^3 + (a+3b)x^2 + (b+3c)x + (c+9)$

Now compare coefficients:

$3a = 3 \Rightarrow a = 1$ $\quad$ $c+9=0 \Rightarrow c=-9$

$a+3b = 16 \Rightarrow b = 5$

And check that $b+3c$ then equals $-22$, as expected!

$3x^3+16x^2-22x = (3x+1)(x^2+5x-9)+9$

You could also do this entirely by comparing coefficients, but using the Remainder Theorem is a nice touch!